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RAN-1183

T.Y.B. Sc. Sem -VI Examination

March / April - 2019

Mathematics Paper : MTH - 601

Ring Theory

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:
☛ **T.Y.B. Sc. Sem -VI**

Name of the Subject :
☛ **Mathematics Paper : MTH - 601**

Subject Code No.: 1 1 8 3

Seat No.:

Student's Signature

(1) All questions are compulsory.
(2) Figures to the right indicate marks of corresponding question.
(3) Follow usual notations.
(4) Use of non-programmable scientific calculator is allowed.

1. Answer the following as directed : (Any FIVE) (10)

(1) R = $\{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}\}$ is a commutative ring under the binary operations $+_{10}$ (addition modulo 10) and x_{10} (multiplication modulo 10). Justify : Every non-zero element in this R has an inverse for x_{10} .

(2) In a ring R; prove that $a.(-b) = -(a.b)$; for all $a, b \in$ R.

(3) Mention all the ideals of the ring J_{13} ; of integers modulo 13.

(4) If U is an ideal of a ring R with a unit element 1 and $1 \in U$, then prove that $U = R$.

(5) Prove that $\bar{3} \mid \bar{5}$ and $\bar{5} \mid \bar{3}$ in the commutative ring J_8 ; of integers modulo 8.

(6) Let R be a Euclidean ring and $a \neq 0$, $b \neq 0$ in R. If b is unit in R, then prove that $d(a) = d(a.b)$.

(7) Justify : $\bar{4}$ and $\bar{8}$ are relatively prime elements in the Euclidean ring J_{11}; of integers modulo 11.

(8) Define a prime element in a Euclidean ring . Which are the prime elements in the Euclidean ring J_7 ; of integers modulo 7 ?

2. Attempt any TWO : (10)

(1) Prove that every finite integral domain is a field.

(2) Prove that the commutative ring D is an integral domain if and only if $a, b, c \in$ D with $a \neq 0$; the relation $a. b = a. c$ implies that $b = c$ holds in D.

(3) Define a Boolean ring. Prove that every Boolean ring is commutative.

3. Attempt any TWO : (10)

(1) Define the Kernel of a homomorphism. Let $\phi : R \rightarrow R'$ be a homomorphism of a ring R into a ring R'. Then prove that: $\phi(0) = 0'$ and $\phi(-a) = -\phi(a)$; for every a in R.

(2) Prove that a homomorphism $\phi : R \rightarrow R'$ of a ring R into a ring R' is an isomorphism if and only if $I(\phi) = (0)$; where $I(0)$ is the Kernel of a homomorphism ϕ .

(3) If R is a commutative ring with a unit element 1 and its only ideals are (0) and R itself, then prove that R is a field.

4. Attempt any TWO : (10)

(1) Define a Euclidean ring. Prove that every field is a Euclidean ring.

(2) Prove that the relation of "*associates*" in a commutative ring R with a unit element is an equivalence relation on R.

(3) Define a greatest common divisor of two elements in *a* commutative ring. Prove that any two greatest common divisors of elements a, b in a Euclidean ring R are associates.

5. Attempt any TWO : (10)

(1) Define relatively prime elements in a Euclidean ring. If a and b are relatively prime elements in a Euclidean R and $a \mid bc$, then prove that $a|c$.

(2) Let R be a Euclidean ring. If $A = (a_0)$ is a maximal ideal of R, then prove that a_0 is a prime element in R.

(3) Define unit in a commutative ring with a unit element. Let R be a Euclidean ring and $a \neq 0$, $b \neq 0$ in R. If b is not unit in R, then prove that $d(a) < d(a. b)$.