RAN-1040

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T.Y.B.Sc. Sem-V Examination

March / April - 2019

Group Theory

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી. **Fill up strictly the details of ☛ signs on your answer book**	Seat No.:
Name of the Examination: ☛ **T.Y.B.Sc. Sem-V**	
Name of the Subject : ☛ **Group Theory**	
Subject Code No.: 1 0 4 0	Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the corresponding questions.

(4) Follow usual notations.

(5) Use of non-programmable scientific calculator is allowed.

Que 1: Answer any FIVE of the following: (10)

1) Find the remainder when 5^{36} divided by 37.

2) True or False: If $a|x$ and $b|x$ then $(a+b)|x$. Justify your answer.

3) If G is a group such that $(a\cdot b)^2 = a^2\cdot b^2$ for all $a, b \in G$, then show that G is abelian.

4) In S_3 give an example of $x, y \in S_3$ such that $(xy)^2 \neq x^2 y^2$

5) If $a \in G$ then define normalizer N(a).

If $G=\left\{\begin{pmatrix} a & b \\ -b & a\end{pmatrix} / a^2+b^2 \neq 0; a,b \in R\right\}$ is a group and $\begin{pmatrix} 2 & 3 \\ -3 & 2\end{pmatrix} \in G$

then find $N\left(\begin{pmatrix} 2 & 3 \\ -3 & 2\end{pmatrix}\right)$.

6) Let $o(G) = 31$. Is G cyclic? Justify your answer.

7) If $o(G) = 24$ and H is a subgroup of G such that $o(H) = 12$. Is H a normal subgroup of G? Justify your answer.

8) Is every subgroup of $Z(G)$ a normal subgroup of G? Justify your answer.

Que 2: Answer any TWO of the following: (10)

1) If a, b are integers, not both 0, then prove that (a, b) exists. Moreover, show that there exists integers m_0 and n_0 such that $(a, b) = m_0 a + n_0 b$.

2) Prove that the relation "congruence modulo n" defines an equivalence relation on the set of integers.

3) To check that n is a prime number, prove that it is sufficient to show that it is not divisible by any prime number p, such that $p \leq \sqrt{n}$.

Que 3: Answer any TWO of the following: (10)

1) If H is any arbitrary non-empty subset of a group G then state and prove the necessary and sufficient condition for H to be a subgroup of G.

2) Let G be the set of all symbols a_i ; $i = 0, 1, 2, \ldots, 6$ where

   $a_i \cdot a_j = ai + j$; if $i + j < 7$ and $a_i \cdot a_j = a_{i+j-7}$ if $i + j \geq 7$.

   Prove that G is an abelian group under the given operation.

3) In a group G, if $a, b \in G$ are any elements and $(ab)^n = a^n b^n$ for three consecutive integers n, then prove that G is abelian.

Que 4: Answer any TWO of the following: (10)

1) State and prove Lagrange's Theorem.

2) Prove that every subgroup of cyclic group is cyclic.

3) Let H be a subgroup of a group G and $a \in G$. If

   $aHa^{-1} = \{aha^{-1} \mid h \in H\}$, then show that aHa^{-1} is a subgroup of G. Also find $o(aHa^{-1})$, if H is finite.

Que 5: Answer any TWO of the following: (10)

1) State and prove fundamental theorem of homomorphism.

2) Let G be the group of integers under addition. Let N be the subgroup of G which contains all the multiples of 5. Find all elements of quotient group G/N. Also find index of N in G.

3) Define: Normal subgroup. Prove that a subgroup N is a normal subgroup of a group G if and only if $gNg^{-1} = N$; for every $g \in G$.
